Statistics

Lecture 7

Feb 19-8:47 AM

Class Quiz 6

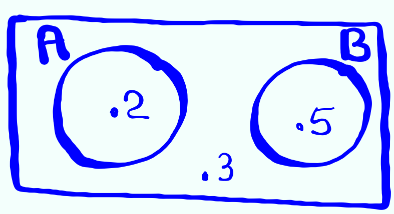
Given $P(A) = .2$, $P(B) = .5$, A & B are mutually exclusive events.

1) Find $P(\bar{A}) = 1 - P(A)$
$= \boxed{.8}$

2) find $P(A \text{ and } B) = \boxed{0}$

3) Find $P(A \text{ or } B)$
$= P(A) + P(B) - P(A \text{ and } B)$
$= .2 + .5 - 0 = \boxed{.7}$

4) Construct its Venn diagram



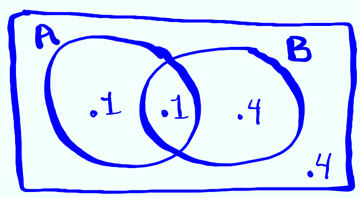
Apr 11-7:57 AM

Suppose $P(A) = .2$, $P(B) = .5$, A and B are independent events.

1) $P(A \text{ and } B) = P(A) \cdot P(B) = (.2)(.5) = \boxed{.1}$

2) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = .2 + .5 - .1$
$= \boxed{.6}$

3) Make Venn Diagram.



4) $P(\overline{A} \text{ and } \overline{B}) = P(\overline{A \text{ or } B}) = 1 - P(A \text{ or } B)$

De Morgan's Law $= 1 - .6 = \boxed{.4}$

5) $P(\overline{A} \text{ or } \overline{B}) = P(\overline{A \text{ and } B}) = 1 - P(A \text{ and } B)$
$= 1 - .1 = \boxed{.9}$

Apr 11-8:13 AM

You are taking a quiz with 3 questions.

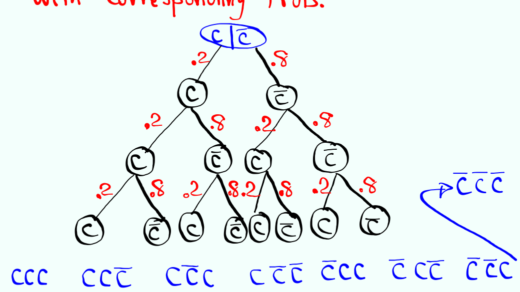
each question has 5 choices, but only one Correct choice.

You are making random guesses.

1) P(guess Correctly on each question)
$= \frac{1}{5} = \boxed{.2}$

2) P(guess incorrectly on each question)
$= \frac{4}{5} = \boxed{.8}$

3) Draw Tree diagram with $C \to$ Correct, $\overline{C} \to$ incorrect. Properly label each branch with Corresponding Prob.



Apr 11-8:19 AM

$P(\text{All Correct}) = P(CCC) = (.2)(.2)(.2) = \boxed{.008}$

$P(\text{All incorrect}) = P(\bar{C}\bar{C}\bar{C}) = (.8)(.8)(.8) = \boxed{.512}$

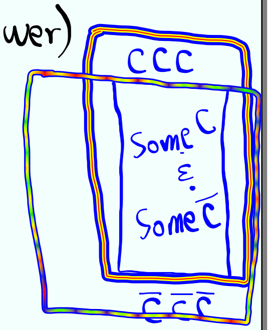
$P(\text{at least 1 Correct answer})$

$= 1 - P(\text{All incorrect}) = 1 - .512 = \boxed{.488}$

$P(\text{at least 1 incorrect answer})$

$= 1 - P(\text{All Correct})$

$= 1 - .008 = \boxed{.992}$



Apr 11-8:29 AM

Multiplication Rule

$P(A \text{ and } B) = P(A) \cdot P(B|A)$

A happens, then

B happens

Given

ex: A box has 3 Red and 7 Blue balls.

Take two balls, no replacement.

$P(RR) = \frac{3}{10} \cdot \frac{2}{9} = \boxed{\frac{1}{15}}$

$P(BB) = \frac{7}{10} \cdot \frac{6}{9} = \boxed{\frac{7}{15}}$

$P(\text{Both balls are Same Color}) = \frac{1}{15} + \frac{7}{15} = \boxed{\frac{8}{15}}$

RR or BB

$P(\text{Both balls are different Color})$

$= 1 - P(\text{Same Color}) = 1 - \frac{8}{15} = \boxed{\frac{7}{15}}$

Apr 11-8:37 AM

4 Females , 8 Males, Select 3 different People

1) $P(\text{All Females}) = P(FFF) = \frac{4}{12} \cdot \frac{3}{11} \cdot \frac{2}{10} = \boxed{\frac{1}{55}}$

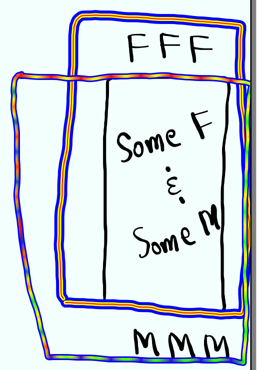
2) $P(\text{All Males}) = P(MMM) = \frac{8}{12} \cdot \frac{7}{11} \cdot \frac{6}{10} = \boxed{\frac{14}{55}}$

3) $P(\text{at least 1 Female})$

$= 1 - P(MMM) = 1 - \frac{14}{55} = \boxed{\frac{41}{55}}$

4) $P(\text{at least 1 male})$

$= 1 - P(FFF) = 1 - \frac{1}{55} = \boxed{\frac{54}{55}}$



Apr 11-8:44 AM

$P(A \text{ and } B) = P(A) \cdot P(B|A)$

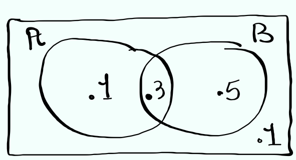
Given

If we isolate $P(B|A)$,

$$\boxed{P(B|A) = \frac{P(A \text{ and } B)}{P(A)}}$$

Conditional Prob.

$P(A) = .4$ $\quad$ $P(B) = .8$ $\quad$ $P(A \text{ and } B) = .3$



$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$

$= \frac{.3}{.4} = \boxed{.75}$

$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$

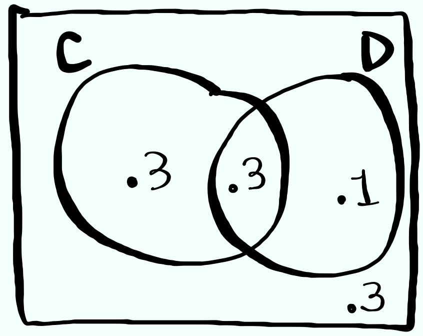
$= \frac{.3}{.8} = \boxed{.375}$

Apr 11-9:04 AM

P(Coffee) = .6

P(Donuts) = .4

P(Coffee and Donuts) = .3



P(Donut | Coffee) = $\frac{P(C \text{ and } D)}{P(C)} = \frac{.3}{.6} = \boxed{.5}$

P(Coffee | Donuts) = $\frac{P(C \text{ and } D)}{P(D)} = \frac{.3}{.4} = \boxed{.75}$

Apr 11-9:10 AM

P(Pants) = .8

P(Shoes) = .5

P(Shoes | Pants) = .75

Find P(Shoes and Pants)

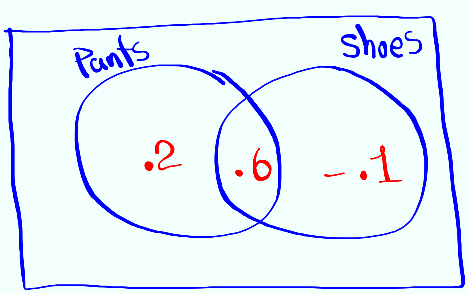
P(Shoes | Pants) = $\frac{P(P \text{ and } S)}{P(P)}$

$.75 = \frac{P(P \text{ and } S)}{.8}$

Cross-Multiply

P(P and S) = (.8)(.75)

$= \boxed{.6}$



This problem is impossible.

Apr 11-9:16 AM

$P(\text{Pants}) = .8$

$P(\text{Shoes}) = .5$

$P(\text{Shoes} \mid \text{Pants}) = .6$

$$P(\text{Shoes} \mid \text{Pants}) = \frac{P(P \text{ and } S)}{P(P)}$$

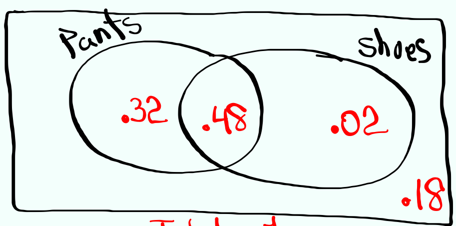
$$.6 = \frac{P(P \text{ and } S)}{.8}$$

Cross-Multiply

Find $P(\text{Shoes and Pants})$.

$.8 - .48 = .32$

$P(P \text{ and } S) = (.6)(.8) = \boxed{.48}$



Total = 1

$.5 - .48 = .02$

SG 13

Apr 11-9:16 AM

Data
1) Qualitative
Non-Numerical
2) Quantitative
Numerical
1) Discrete
Countable
2) Continuous
Measureable

Apr 11-9:31 AM

Let x be a discrete random variable with prob. dist. of $P(x)$.

what is prob. dist.?

It is a way to give/find prob. of all possible outcomes in the Sample Space.

what is a Sample Space?

It is a complete list of all Possible outcomes.

Prob. dist could be

1) in the form of a chart
2) in the form of a graph
3) in the form of some formula
4) using just def. of probabilities

Apr 11-9:33 AM

Some rules

1) $0 \leq P(x) \leq 1$
2) $\sum P(x) = 1$
3) $P(x) = 1 \iff$ Sure event
4) $P(x) = 0 \iff$ Impossible event
5) $0 < P(x) \leq .05 \iff$ Rare event

Apr 11-9:40 AM

Consider the chart below

x	$P(x)$
1	.2
2	.5
3	.3

1) $\sum P(x) = .2 + .5 + .3 = 1$ ✓

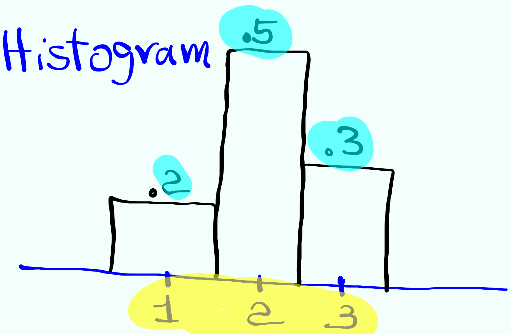
2) $P(x \geq 2) = .5 + .3 = .8$

3) $P(x \leq 2) = .5 + .2 = .7$

4) Draw Prob. Dist. Histogram

$x \to$ Midpoint

$P(x) \to$ Rel. F.



Apr 11-9:43 AM

Consider the chart below

x	$P(x)$
1	.2
2	.3
3	.4
4	.1

1) Find $P(x=4)$

$= 1 - [.2 + .3 + .4]$

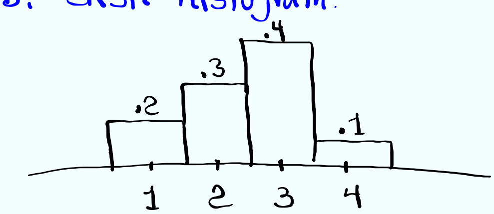
total Prob.

$= 1 - .9 = \boxed{.1}$

2) $P(x=1 \text{ or } x=4) =$

$.2 + .1 = \boxed{.3}$

3) Draw Prob. dist. histogram.



Apr 11-9:48 AM

Complete the chart below

x	$P(x)$	$x \cdot P(x)$	$x^2 \cdot P(x)$
1	.2	.2	.2
2	.5	1.0	2.0
3	.3	.9	2.7

1) $\sum P(x) = 1$ ✓

2) $\sum x\,p(x) = 2.1$

3) $\sum x^2 p(x) = 4.9$

4) Find $\sum x^2 p(x) - \left(\sum x\,p(x)\right)^2 = 4.9 - 2.1^2 = \boxed{.49}$

5) Find $\sqrt{\text{last answer}} = \sqrt{.49} = \boxed{.7}$

Apr 11-9:54 AM

Mean μ "mu"

Variance σ^2 "Sigma^2"

Standard Deviation σ "Sigma"

$$\boxed{\begin{aligned} \mu &= \sum x\,p(x) \\ \sigma^2 &= \sum x^2 p(x) - \mu^2 \\ \sigma &= \sqrt{\sigma^2} \end{aligned}}$$

68% Range $\rightarrow \mu \pm \sigma$

95% Range $\rightarrow$ Usual Range $\rightarrow \mu \pm 2\sigma$

99.7% Range $\rightarrow \mu \pm 3\sigma$

Apr 11-10:12 AM

How to use TI to find μ, σ, and σ^2:

$x \to L1$

$P(x) \to L2$

$\boxed{\text{STAT}}\ \boxed{\to}$ CALC

$\boxed{\text{1: 1-Var Stats}}$

x	$P(x)$
1	.2
2	.5
3	.3

$\mu = \bar{x} = 2.1$

$\sigma = \sigma_x = .7$

$n = 1$

List : L1

FreqList : L2

$\boxed{\text{Calculate}}$

$\boxed{\text{VARS}}\ \boxed{\text{5: Statistics}}\ \boxed{4: \sigma_x}\ \boxed{x^2}$

$\boxed{\text{Enter}}\ \boxed{.49} \leftarrow \sigma^2$

Apr 11-10:16 AM

Consider the chart below

x	$P(x)$
1	.15
2	.25
3	.45
4	.15

$x \to L1$, $P(x) \to L2$

$\boxed{\text{STAT}}\ \boxed{\to}$ CALC

$\boxed{\text{1: 1-Var Stats}}$

List : L1

FreqList : L2

$\boxed{\text{Calculate}}$

$\sigma^2 = \frac{21}{25}$

$\mu = \bar{x} = 2.6$

$\sigma = \sigma_x = .917$

$n = 1$

Find σ^2 in reduced fraction

$\boxed{\text{VARS}}\ \boxed{\text{5: Statistics}}\ \boxed{4: \sigma_x}$

$\boxed{x^2}\ \boxed{\text{Math}}\ \boxed{\text{1: ▸Frac}}\ \boxed{\text{Enter}}$

Apr 11-10:22 AM

2 quarters, 3 nickels, take 2 coins, no replacement.

Sample Space
- NN → Total 10¢ — $P(10¢) = \frac{3}{5} \cdot \frac{2}{4} = \frac{6}{20}$
- NQ, QN → Total 30¢ — $P(30¢) = 2 \cdot \frac{3}{5} \cdot \frac{2}{4} = \frac{12}{20}$
- QQ → Total 50¢ — $P(50¢) = \frac{2}{5} \cdot \frac{1}{4} = \frac{2}{20}$

Total	P(Total)
10¢	6/20
30¢	12/20
50¢	2/20

Total → X → L1

P(Total) → P(X) → L2

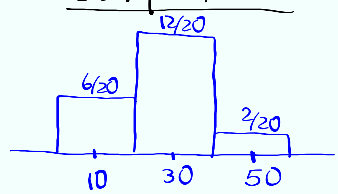
Use 1-Var Stats with L1 & L2

$\mu = 26$

$\sigma = 12$

$n = 1$

$\sigma^2 = 144$



Apr 11-10:28 AM

Expected Value

$\mu = \bar{x}$

Net	P(Net)
\$10 − \$100	1/20
\$10 − 0	19/20

Net → L1

P(Net) → L2

Expected Value Per ticket $= \mu = \bar{x} = \boxed{\$5}$

20 students

20 tickets were sold

each ticket \$10

one ticket is randomly drawn

winner gets a calc worth \$100.

I make \$5 per ticket sold.

Apr 11-10:38 AM

You buy luggage insurance from the airline.
You pay $100, Any damages, Airline Pays You $1000.

$P(\text{Any damage}) = .1\% = .001$

Net	P(Net)	
100 - 1000	.001	Damage
100 - 0	.999	$\overline{\text{Damage}}$

Net → L1

P(Net) → L2

Expected Value Per Policy Sold

$\mu = \bar{x} = \$99$

Apr 11-10:45 AM

Draw one card from a full standard deck of playing cards. You must pay $5 to play.

If You draw	I give You
Ace	$50
Red face	$10
Any other Card	$0

Expected Value per draw for the house

Net	P(Net)	
5 - 50	4/52	Ace
5 - 10	6/52	Red face
5 - 0	42/52	Any other Card

$E.V. = \mu = \bar{x} = 0$

$n = 1$

$\boxed{\text{SG 14 \& 15}}$ ✓

Apr 11-10:49 AM

Class QZ 7

Consider the Chart below

x	$P(x)$
1	.05
2	.15
3	.25
4	.35
5	.20

Find

1) $\mu = 3.5$
2) $\sigma = 1.1$
3) $n = 1$
4) $\sigma^2 = \frac{5}{4}$

} Round to 1-dec.

} Reduced fraction

$x \to L1$, $P(x) \to L2$

1 - Var Stats

Apr 11-10:58 AM